Designing a Multi-Period Water Distribution Network with a Hybrid Simulated Annealing

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Abstract. The design optimization of water distribution systems, considering multi-period restrictions with time varying demand patterns, is a complex optimization problem present in modern cities. Taking into account this issue, we propose a hybrid optimization technique based on Simulated Annealing (HSA) enhanced with a local search procedure, in order to obtain good quality networks designs. The SA performance is very sensitive to the choice of the cooling scheme, which includes parameters as the cooling function and the initial temperature. For this reason, we have analysed the HSA behaviour using proportional, exponential, logarithmic and a new random cooling functions and three different initial temperatures. An extensive experimentation using different benchmark networks has been carried out to test our proposals. Moreover, a comparison with an approach from the literature has revealed the goodness to solve this network design problem.

Keywords: Water Distribution Network Design, Optimization, Simulated Annealing, Cooling Schedule

1 Introduction

Water is one of the most important natural resources, being an essential commodity for human life. Therefore, a safe, adequate, and accessible supply of potable water is one of the basic necessities in any modern city. A distribution system therefore consists of pipe lines of various sizes for carrying water, valves for controlling the flow, service connections to the individual homes, and distribution reservoirs for storing the water to be fed into the distribution pipes. Therefore, the design of a water distribution network plays an important role. The solution concerning the layout, design, and operation of the network of pipes should result from good planning and management procedures. In this way, this problem known as Water Distribution Network Design (WDND), requires to manage an important number of variables (pipes, pipe diameters, demand nodes, water pressure, reservoirs, etc.), and constraints (water velocity, pressure, etc.). Consequently, even for simple networks, this problem is very difficult to solve, in particular it is classified as NP-hard [1].

The first research works regarding the optimization of WDND were concerned with the single-period, single-objective, gravity-fed design optimization problem. These preliminary works applied linear programming [2, 3], and nonlinear programming [4, 5]. Further research proposed metaheuristics to solve these problems. In these area, trajectory-based proposals were applied, such as: Simulated Annealing [6–8] and Tabu Search [9]. Population-based metaheuristics were also considered, for example, Genetic Algorithms [10–12], Ant Colony Optimization [13, 14], Scatter Search [15], and Differential Evolution [16].

Recently, more complex and realistic formulations were considered, such as the extension to a multi-period setting (time varying demand patterns). Farmani et al. [17] formulated the design problem as a multi-objective optimization problem and applied a multi-objective evolutionary algorithm. In [11], a genetic algorithm was used to solve six small instances, which also incorporated the velocity constraint on the water flowing through the distribution pipes. This constraint was also taken into account in [18], but the authors used mathematical programming on bigger, closer-to-reality instances. A Differential Evolution (DE) algorithm was proposed in [19] to minimize the cost of the water distribution network. Another version of a DE algorithm to solve this problem was presented in [20]. An Iterative Local Search [21] was specifically-designed in order to consider that every demand node has 24 hrs water demand pattern and a new constraint, which imposes a limit on the maximal velocity of water through the pipes.

In this work, we have designed a hybrid Simulated Annealing (HSA) to improve and optimize the distribution network design considering time varying demand patterns and the maximum water velocity constraint [21]. This HSA enhances the layout of the network by using a local search technique. The methodology devised in this work characterizes the HSA behaviour regarding the main control algorithmic parameter (temperature), and it is summarized as follows. We evaluate HSA with four different schemes to schedule the cooling process. Moreover, we consider three initial temperatures for each variant from low to high values. This work is based on a previous research [22], incorporating the analysis of the temperature impact on the HSA's performance. We test the performance of our proposals with a set of networks with different sizes expressed by number of pipes and characteristics. We then will compare the HSA algorithm with the literature. The planned analyses and comparisons will help us to claim that our proposals outperform the state-of-the-art. In order to get some meaningful conclusions, we present some studies considering relevant aspects such as cost values, HSA's behavior, and computational effort.

The remainder of this article is structured as follows. Section 2 introduces the problem definition. Section 3 explains our algorithmic proposal, HSA, to solve the WDND optimization problem and the HSA's variants. Section 4 describes the experimental analysis and the methodology used. Then, the sections 5 and 6 present the result analysis of the variants and the comparison with an ILS [21] from the literature, respectively. Finally, Section 7 summarizes our conclusions and sketches our future work.

2 Multi-Period Water Distribution Network Design

The objective of the WDND problem is to minimize the total investment cost (TIC) in a water distribution network design. The problem can be characterized as: simple-objective, multi-period, and gravity-fed. Two restrictions are considered: the limit of water speed in each pipe and the demand pattern that varies in time. The network can be modeled by a connected graph, which is described by a set of nodes $N = \{n_1, n_2, ...\}$, a set of pipes $P = \{p_1, p_2, ...\}$, a set of loops $L = \{l_1, l_2, ...\}$, and a set of commercially available pipe types $T = \{t_1, t_2, ...\}$. The TIC is obtained by the formula shown in Equation 1,

$$\min TIC = \sum_{p \in P} \sum_{t \in T} L_p IC_t x_{p,t} \tag{1}$$

where IC_t is the cost of a pipe p of type t, L_p is the length of the tube, and $x_{p,t}$ is the binary decision variable that determines whether the tube p is of type t or not. The objective function is limited by: physical laws of mass and energy conservation, minimum pressure demand in the nodes, and the maximum speed in the pipes, for each time $\tau \in \mathcal{T}$. These laws are explained in the following paragraphs.

Mass conservation law: It must be satisfied for each node N in each period of time τ. This law establishes that the volume of water flowing towards a node in a unit of time must be equal to the flow that leaves it (see Equation 2),

$$\sum_{n_1 \in N/n} Q_{(n_1,n),\tau} - \sum_{n_2 \in N/n} Q_{(n,n_2),\tau} = WD_{n,\tau} - WS_{n,\tau} \quad \forall n \in N \quad \forall \tau \in \mathcal{T} \tag{2}$$

where $Q_{(n_1,n),\tau}$ is the flow from node n_1 to node n at time τ, $WS_{n,\tau}$ is the external water supplied and $WD_{n,\tau}$ is the external water demanded.

Energy conservation law: It states that the sum of pressure drops in a closed circuit in an instant of time τ is zero. These drops can be approximated using the Hazen-Williams equations with the parameters used in EPANET 2.0 [23] (the hydraulic solver used in this paper), as indicated in Equation 3.

$$\sum_{p \in l} \left[\frac{10.6668 y_{p,\tau} Q_{p,\tau}^{1.852} L_p}{\sum_{t \in T} (x_{p,t} C_t^{1.852} D_t^{4.871}} \right] = 0 \quad \forall l \in L \quad \forall \tau \in \mathcal{T} \tag{3}$$

In Equation 3, $y_{p,\tau}$ is the sign of $Q_{p,\tau}$ that indicates changes in the water flow direction relative to the defined flow directions, $Q_{p,\tau}$ is the amount of water flowing through pipe p in time τ, L_p is the pipe length, C_t is the Hazen-Williams roughness coefficient of pipe type t, and D_t is the diameter of pipe type t.

Minimum pressure head requirements: for each node n in each period of time τ, it must be satisfied (see Equation 4),

$$H_{n,\tau}^{min} \leq H_{n,\tau} \quad \forall n \in N \quad \forall \tau \in \mathcal{T} \tag{4}$$

being H^{min} the minimum node pressure and $H_{n,\tau}$ the node's current pressure.

Maximum water velocity: The water velocity $v_{p,\tau}$ can not exceed the maximum stipulated speed $v_{p,\tau}^{max}$. Equation 5 shows this relationship.

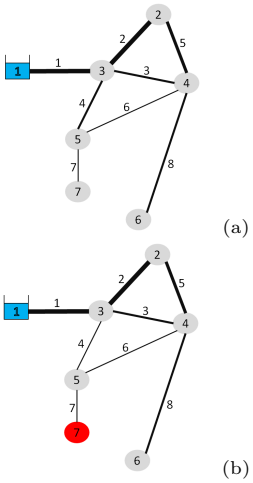
$$v_{p,\tau} \leq v_{p,\tau}^{max} \quad \forall p \in P \quad \forall \tau \in \mathcal{T} \tag{5}$$

3 HSA for the Multi-Period WDND Problem

Simulated Annealing (SA), proposed by Kirkpatrick et al. [24], is a simple and efficient trajectory-based metaheuristic. SA is based on the principles of statistical thermodynamics, which models the physical process of heating a material and then slowly lowering the temperature to decrease defects, thus minimizing the system energy. In a similar way, at each virtual annealing temperature, the SA algorithm generates a new potential solution (or neighbour of the current state) to the problem considered by altering the current state, according to a predefined criterion. The acceptance of the new state is then based on the satisfaction of the Metropolis criterion, and this procedure is iterated until convergence.

At the beginning (with a high temperature), SA accepts solutions with high cost values under a certain probability in order to explore the search space and to escape from local optima. During the annealing process this probability decreases according to temperature cooling; intensifying the search and reducing the exploration in order to exploit a restricted area of the search space.

SA evolves by a sequence of transitions between states and these transitions are generated by transition probabilities. Consequently, SA can be mathematically modeled by Markov chains, where a sequence of chains is generated by a transition probability that is calculated involving the current temperature.



(a)

(b)

Pipe ID	Length (m)
1	31
2	20
3	35
4	37
5	24
6	50
7	12
8	65

(c)

Diam. (mm)	Roughness unitless	Cost
60	130	22
80	130	35
100	130	38
150	130	50
200	130	61

(d)

Fig. 1. Different solutions or network designs. (a) Solution 1; (b) Solution 2; (c) Pipe lengths; (d) Available pipe types with their corresponding costs.

Table 1. Different solutions or network designs in vector representation.

Solution	Pipe ID	1	2	3	4	5	6	7	8	Feasibility
	Length (m)	**31**	**20**	**35**	**37**	**24**	**50**	**12**	**65**	**TIC**
1	diam. (mm)	150	150	80	80	100	60	60	80	feasible
	cost	1550	1000	1225	1295	912	1100	264	2275	9621
2	diam. (mm)	150	150	80	60	100	60	60	80	infeasible
	cost	1550	1000	1225	814	912	1100	264	2275	9140

The proposal consists in adapting and hybridizing the SA algorithm to solve the Multi-Period WDND optimization problem. Furthermore, different mechanisms to control the temperature cooling are incorporated to the Hybrid Simulated Annealing (HSA) algorithm. Although in [7] the authors also proposed an SA for two small instances of the WDND problem, in this work we solve a more complex and realistic version of this problem with a SA hybridised, using fifty large and diverse network configurations. Another difference lies in using other mechanisms to control the temperature cooling.

The first design issue is to define the representation of a solution. In this case, a solution to the WDND problem is a network, as shown in Figure 1 (a) and (b), which is represented by a vector. Each vector element is the diameter selected for the pipe it represents. Table 1 shows the vectors that correspond to the candidate solutions in Figure 1(a) and (b). The total investment cost for each solution is calculated by the Equation 1, using the input data from tables (c) and (d) of Figure 1. The first solution is hydraulically feasible (satisfying all constraints mentioned in Section 2) and the second one is infeasible (violating the minimum pressure constraint in node 7).

Algorithm 1 HSA Algorithm to solve the WDND optimization Problem

```
1: k = 0;
2: initTemp(T)
3: initialize(S_0);
4: TIC_0 = evaluate(S_0);
5: repeat
6:    repeat
7:       k = k + 1;
8:       S_1 = MP-GRASP_LS(S_0);
9:       TIC_1=evaluate(S_1);
10:      if TIC_1 < TIC_0 then
11:         S_0 = S_1;
12:         TIC_0 = TIC_1
13:      end if
14:      generate S_2 = perturbation_operator(S_0);
15:      TIC_2 = evaluate(S_2);
16:      if (TIC_2 < TIC_0) or (exp^((TIC_2 - TIC_0)/T) > random(0,1)) then
17:         S_0 = S_2;
18:         TIC_0 = TIC_2
19:      end if
20:   until (k mod MCL) == 0
21:   update(T);
22: until stop criterion is met
23: return S_0;
```

The pseudo-code of the HSA algorithm proposed to solve the WDND optimization problem is shown in Algorithm 1. HSA begins with the initialization of the temperature (line 2). After that, HSA generates a feasible initial solution S_0 applying both `HighCost` and `Lowcost` mechanisms proposed in [21] (line 3), which is then evaluated (line 4). For the last process, HSA uses the EPANET 2.0 toolkit [23] to solve the hydraulic equations, since this hydraulic solver is applied in most existing works. Once the initialization process ends, an iterative process starts (lines 5 to 22). As a first step in the iteration, the hybridization is carried out in order to intensify the search into the current region of the solution space. In this way a feasible solution, S_1, is obtained by applying the MP-GRASP local search [21] to S_0 (line 8), and then a greedy selection mechanism is performed (lines 10-13). As a consequence, S_0 can be replaced by S_1 if this is better than S_0. In the next step, a perturbation operator is used to obtain a feasible neighbor, S_2, from S_0 (line 12), in order to explore another areas of the search space. This perturbation randomly changes some pipe diameters. If S_2 is worse than S_0, S_2 can be accepted under the Boltzmann probability (line 16, second condition). In this way, at high temperatures (T) the exploration of the search space is strengthened. In contrast, at low temperatures the algorithm only exploits a promising region of the solution space, intensifying the search. In order to update T, a cooling schedule is used (line 21) and it is applied after a certain number of iterations (k) given by the Markov Chain Length (MCL) (line 20). Finally, SA ends the search when the total evaluation number is reached or the $T0$.

Most of the HSA search components are fixed in function of the problem to be solved. Consequently, the search space, cost (evaluation) function, perturbation operator, and local search are directly related to the WDND problem. The main search component to be defined is the temperature and their annealing schedule. Therefore one of the most important issues in HSA is how to control the annealing or cooling process so that the system cools gradually from a higher temperature, ultimately freezing to a global minimum state. Many attempts have been made to derive or suggest good schedules [25]. At the same level of importance, the choice of the right initial temperature is also crucial, which must not be too high to conduct a random search for a period of time but high enough to allow moves to almost neighborhood state. In this work, we propose to analyse the impact on the success of the HSA imposed by the appropriate setting of the initial temperature and the cooling scheme, which are related with the problem nature and the values of the objective function.

3.1 HSA's Variants: Cooling Scheme

This section describes the different HSA's variants which include the most known cooling process in the literature and a new one proposed in this work, which are identified as $\text{HSA}_{coolingprocess}$.

The first HSA's variant applies the proportional cooling scheme, also called geometric schedule [24], which is denominated HSA_{Prop}. The temperature is updated using the Equation 6:

$$T_{k+1} = \alpha * T_k \tag{6}$$

where α is a constant close to, but smaller than, 1. Particularly, we calculate α as follows:

$$\alpha = \frac{k}{k+1} \tag{7}$$

This scheme is the most popular cooling function, since the temperature decay is not too slow neither too fast allowing to achieve an equilibrium between exploitation and exploration.

HSA_{Exp} uses the exponential cooling scheme [24] to produce the temperature decay. The Equation 8 describes these process, where the constant $\alpha^k < 1$ is calculated as shown in the Equation 9. This schedule quickly cools the temperature reducing the required time and iterations to converge to a good solution. In big and complex problems, this becomes in a disadvantage, given that the equilibrium between the exploitation and exploration is broken.

$$T_{k+1} = T_k * \alpha^k \tag{8}$$

$$\alpha^k = \frac{e^k}{e^{1+k}} \tag{9}$$

HSA_{Log} employs the logarithmic cooling scheme [26], which modifies the temperature, as shown in the Equation 10. In this Equation, the chain converges to a global and minimal energy value, where the constant C is computed as the Equation 11 shows. This schedule is too slow to be applied in practice but has the property of the convergence proof to a global optimum [27].

$$T_{k+1} = C * T_k \tag{10}$$

$$C = \frac{ln(k)}{ln(1+k)} \tag{11}$$

Furthermore, we propose a fourth HSA's variant, named HSA_{Rand}. This new variant combines the three previous explained cooling schemes in only one schedule process. In each iteration, HSA_{Rand} randomly selects one of these schemes in order to reduce the temperature. In this way, we try to enhance HSA by aggregating the advantages of these three schemes and mitigating their disadvantages.

3.2 HSA's Variants: Initial Temperature

As previously mentioned, the choice of the right initial temperature has an important role in the HSA performance to find good solutions. A classical and intuitive method is used in this work as described in Kirkpatrick et al. [24]. It consists in computing a temperature such that the acceptance ratio is approximately equal to a given value χ_0. Given a T_s seed temperature, the initial temperature is computed by the procedure shown in Algorithm 2. The output, T_0, is determined such that, when applying the Boltzmann criterion, worse solutions are accepted with a high probability value. To achieve this, the algorithm starts from a T_s that is increased until the aforementioned acceptance is reached.

Algorithm 2 Pseudocode of algorithm for setting initial temperature T0

```
1: function initTemp (Ts)
2: T_0 = T_s;
3: initialize(S_1);
4: TIC_1=evaluate(S_1);
5: while acceptability rate is not reached do
6:    update T_0 ;
7:    S_2 = S_1;
8:    TIC_2=evaluate(S_2);
9:    for i=0 to test do
10:      S_2 = MP-GRASP_LS(S_1);
11:      TIC_2=evaluate(S_2);
12:   end for
13: end while
14: return T0
```

In this work, we propose to study the impact of using three different T_s in the performance of the different HSA's variants proposed. In particular, T_s takes values in $\{1, 100, 1000\}$, from small to large seeds, which are very dissimilar seeds allowing a different number of HSA iterations of the main loop. These algorithms are identified by incorporating T_s in the name of the HSA's variant, for example $\text{HSA}_{Rand100}$ means HSA_{Rand} variant with T_s set to 100.

4 Experiments with HSA's Variants

This section explains the design of experiments followed in this work so that our conclusions could be both objective and valid. In order to evaluate the HSA's variants, the HydroGen instances of WDND optimization problem [28] are solved. These instances arise from 10 different distribution networks, named as HG-MP-i with $i \in \{1, 10\}$ (see Table 2 for more specifications). A set of 16 different pipe types is used and their characteristics and costs can be found in Table 3. The demand nodes are divided into five categories (domestic, industrial, energy, public services, and commercial demand nodes), each one with a corresponding base load and demand pattern[3]. In this way, each HG-MP-i network consists of five different instances, totalling 50 instances.

In our experiments, we use four different types of cooling schemes (see Section 3.1) and also T_s seed temperature (in Section 3.2), yielding to 12 different HSA's variants (HSA_{PropT_s}, HSA_{ExpT_s}, HSA_{LogT_s}, and HSA_{RandT_s} with $T_s \in \{1, 100, 1000\}$). We consider an acceptance probability χ_0 of 0.8. The stop condition of the HSA's variants is to reach 100.000 evaluations of the objective function. We performed 30 independent runs of each instance because of the stochastic nature of the algorithms, in order to gather meaningful experimental data and apply statistical confidence metrics to validate our results and conclusions. As a result, a total of 18000 executions ($50 \times 4 \times 3 \times 30$) were carried out. Before performing the statistical tests, we first checked whether the data followed a normal distribution by applying the Shapiro-Wilks test. Where the data was distributed normally, we later applied an ANOVA test. Otherwise, we used

[3] The base loads can be found in the EPANET input files of the instances

Table 2. Information on the HydroGen networks.

Network	Meshedness	Pipes	Demand	Water
HG-MP-1	0.2	100	73	1
HG-MP-2	0.15	100	78	1
HG-MP-3	0.1	99	83	1
HG-MP-4	0.2	198	143	1
HG-MP-5	0.15	200	155	1
HG-MP-6	0.1	198	166	1
HG-MP-7	0.2	299	215	2
HG-MP-8	0.15	300	232	2
HG-MP-9	0.1	295	247	2
HG-MP-10	0.2	397	285	2

Table 3. Available pipe types and their corresponding costs.

Number	Diam. (mm)	Roughness	Cost	Number	Diameter	Roughness	Cost
1	20	130	15	9	200	130	116
2	30	130	20	10	250	130	150
3	40	130	25	11	300	130	201
4	50	130	30	12	350	130	246
5	60	130	35	13	400	130	290
6	80	130	38	14	500	130	351
7	100	130	50	15	600	130	528
8	150	130	61	16	1,000	130	628

the KruskalWallis (KW) test. This statistical study allows us to assess whether or not there were meaningful differences between the compared algorithms with a confidence level of 99%.

5 Analysis of the Results Obtained by our Proposals

In this section, we summarize and analyse the results of using the 12 proposed HSA's variants for all the WDND problem instances, following the next methodology. First, we analyse the behavior of these variants considering the results shown in the tables 4, 5, and 6, taking the different T_s into account, respectively. The columns 2-5 show the average of the best TIC values found by the four variants for the 50 instances grouped by their corresponding distribution network. The minimal TIC values found by each group are boldfaced. In the last column, the results of the KW test are summarized, where the symbol "+" indicates that the behavior of the four HSA's variants are statistically similar, while the symbol "-" specifies that these behaviors are significantly different. Secondly, we analyse the computational effort for each HSA proposed, considering the total time execution time (measured in seconds) of the search, as shown in the Figure 2.

Analysing the results from Table 4, a similar statistical behavior between the HSA's variants with $T_s = 1$ is observed (colomun 6), although HSA_{Prop1} finds the minimal TIC values in the 60% of the networks. HSA_{Rand1} obtains the minimum in a 20% of the networks, whereas the remaining HSA's variants in a 10% each one. From the study of the results shown in Table 5, the significant statistical differences between the HSA algorithms are present. In this case, the

Table 4. Averages of the best TIC values found by each HSA's variant with $T_s = 1$.

Network	HSA_{Prop1}	HSA_{Exp1}	HSA_{Log1}	HSA_{Rand1}	KW
HG-MP-1	**340872.6**	342077.0	342065.8	341584.8	+
HG-MP-2	302643.2	314129.6	307264.8	**301988.2**	+
HG-MP-3	**392959.4**	396474.8	396860.2	392993.4	+
HG-MP-4	**764435.8**	783772.8	778993.6	766045.2	+
HG-MP-5	773268.0	**767347.6**	773764.4	782044.2	+
HG-MP-6	**792741.4**	808084.6	810206.4	803241.0	+
HG-MP-7	**910839.2**	953582.8	926824.2	918531.2	+
HG-MP-8	978102.0	1008866.6	**966027.2**	969844.4	+
HG-MP-9	**928715.6**	951837.8	943209.2	929009.2	+
HG-MP-10	925406.2	949786.4	917229.2	**910357.2**	+

Table 5. Averages of the best TIC values found by each HSA's variant with $T_s = 100$.

Network	$\text{HSA}_{Prop100}$	HSA_{Exp100}	HSA_{Log100}	$\text{HSA}_{Rand100}$	KW
HG-MP-1	340965.4	342736.2	342136.8	**339255.0**	-
HG-MP-2	302986.2	304191.2	302875.0	**302693.6**	-
HG-MP-3	393433.4	393408.2	**392823.4**	395317.6	-
HG-MP-4	763320.6	768556.0	762387.8	**759030.6**	-
HG-MP-5	777919.0	**769613.2**	774843.0	774762.2	-
HG-MP-6	790831.4	793095.6	783136.4	**774950.0**	-
HG-MP-7	938746.4	915456.6	**899668.6**	903309.4	-
HG-MP-8	**951530.6**	987936.0	955793.6	989918.8	-
HG-MP-9	931709.2	929255.8	930488.2	**919305.6**	-
HG-MP-10	912964.4	918225.8	913257.4	**902029.0**	-

Table 6. Averages of the best TIC values found by each HSA's variant with $T_s = 1000$.

Network	$\text{HSA}_{Prop1000}$	$\text{HSA}_{Exp1000}$	$\text{HSA}_{Log1000}$	$\text{HSA}_{Rand1000}$	KW
HG-MP-1	**341634.4**	345761.2	345582.2	343972.0	+
HG-MP-2	**303277.2**	307080.0	303702.6	303988.4	-
HG-MP-3	399776.8	403734.2	**399038.8**	400959.2	-
HG-MP-4	771927.0	796273.0	771515.0	**766651.6**	-
HG-MP-5	**770892.2**	781893.2	773441.8	781434.8	-
HG-MP-6	786640.2	809303.8	789264.0	**795116.2**	-
HG-MP-7	919760.2	965366.4	**905722.2**	920034.8	-
HG-MP-8	987764.0	1014962.0	960103.2	**952484.0**	-
HG-MP-9	933601.4	954523.0	918491.2	**907094.6**	-
HG-MP-10	**908888.2**	949537.4	917604.4	929312.0	-

$\text{HSA}_{Rand100}$ finds the best TIC values in the 60% of the networks, followed by HSA_{Log100} with the 20%. A poor performance can be observed in $\text{HSA}_{Prop100}$ and HSA_{Exp100}. When the $T_s = 1000$ is considered, the KW results indicate that the behavior of all HSA variants is statistically different for almost every network. Both $\text{HSA}_{Prop1000}$ and $\text{HSA}_{Exp1000}$ obtain the best solutions in 40% of the networks each one, the remaining 20% of the best TIC values are achieved by $\text{HSA}_{Log1000}$. Finally, when the boldfaced values of these tables are compared, a relevant result is inferred because when $T_s = 100$ the best network designs are found in the 80% of the networks.

Regarding the computational effort point of view, the Figure 2 shows the HSA's execution times (in seconds) to carry out the maximum number of evaluations for each test case, grouped by distribution network. Similar execution times are observed when the HSA's variants initialize the temperature with

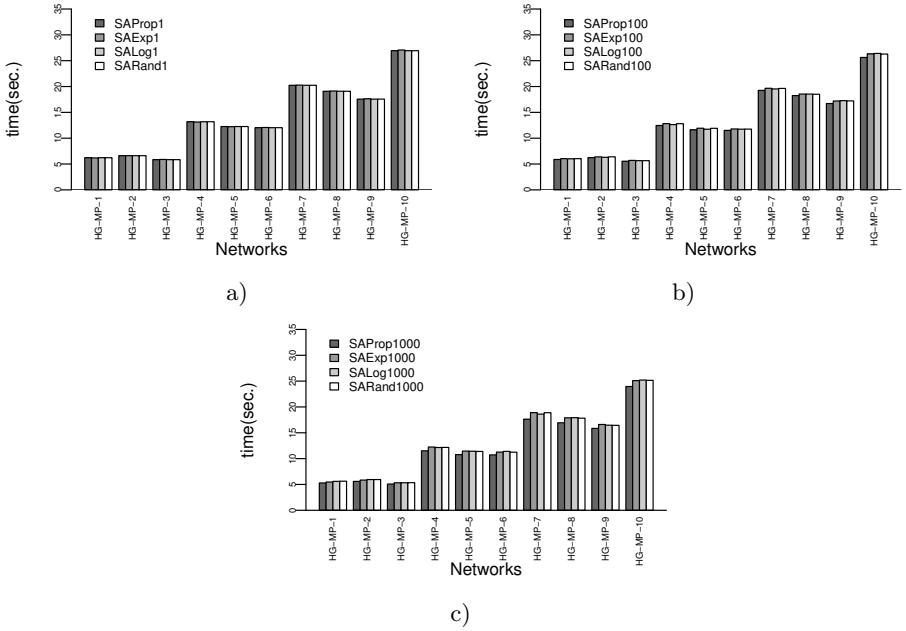


Fig. 2. Average total search time of HSA's variants for each T_s. a) T_s=1, b) T_s=100, and c) T_s=1000.

$T_s = 1$. However, differences in the total execution time are observed when T_s is set to the remaining values. Being the $\text{HSA}_{Prop_{100}}$ and $\text{HSA}_{Prop_{1000}}$ the fastest variants for the search, whereas the $\text{HSA}_{Log_{100}} and \text{HSA}_{Log_{1000}}$ are the slowest ones. These observations are supported by KW tests with $\alpha = 0.01$.

Analysing the network specifications (see Table 2) and the execution times together, we observe that the number of pipes and demand nodes influence the time spent by the HSA's variants. In this way, three groups of three networks can be formed exhibiting similar execution times, except the HG-MP-10 network that is out of these sets. The grouped instances have consecutive numbers, e.g. the set of the HG-MP-1, 2, and 3 networks have similar number of pipes and demand nodes, and so on. Furthermore, analysing what happened into each set, the following behavior is detected. The HSA's variants that design networks with more demand nodes spend less execution time than when they solve the other two cases. This happen because more feasible networks can be designed and, consequently, HSA needs less time to find one of them. Finally, the HG-MP-10 network is the most expensive case to solve, due to it is the biggest instances considering the number of pipes and demand nodes.

Summarizing, HSA_{Rand} under each T_s value is a very good choice to solve the WDND problem. The reason of this is that HSA_{Rand} finds more times the best known TIC values than the remaining variants, spending a reasonable ex-

Table 7. The best known TIC values found by our proposals and ILS.

Network	Best Known TIC	Alg.
HG-MP-1	339255.0	$\mathbf{HSA}_{Rand100}$
HG-MP-2	301988.2	$\mathbf{HSA}_{Rand1}$
HG-MP-3	392823.4	$\mathbf{HSA}_{Log100}$
HG-MP-4	752400.0	**ILS**
HG-MP-5	762000.0	**ILS**
HG-MP-6	774950.0	$\mathbf{HSA}_{Rand100}$
HG-MP-7	899668.6	$\mathbf{HSA}_{Log100}$
HG-MP-8	951530.6	$\mathbf{HSA}_{Prop100}$
HG-MP-9	907094.6	$\mathbf{HSA}_{Rand1000}$
HG-MP-10	902029.0	$\mathbf{HSA}_{Rand100}$

ecution time. Furthermore, the HSA's runtime is affected by the growing and combination of the number of pipes and demand nodes.

6 Comparison of HSA's Variants with the Literature

The performance of the HSA's variants are compared with the ILS proposed in [21]. This metaheuristic is chosen from literature for this purpose, since its authors also used the HydroGen instances to test it. To ensure a fair comparison, both algorithms use the same stop criterion that is set in 100,000 Epanet calls. In this way, our results can be contrasted against ones of the state-of-the-art, allowing to know the level of quality reached by our proposals.

In Table 7, the best known TIC values for the 50 instances, grouped by their corresponding distribution network, are shown. Additionally, the algorithm that found each value is specified. Analysing this table, better TIC values are found when they are solved by HSA in the 80% of the problem instances.

On the one hand, the HSA_{Rand} is the variant that finds the best known TIC values in the 50% of the distribution networks. The advantage of this variant arises out from the combination of the three traditional cooling schemes in only one schedule process.

On the other hand, when the focus is on the seed of the initial temperature, the HSA's variants with $T_s = 100$ achieve the best results. This achievement is a consequence of the initial temperature is not too high to carry out a random search for a long period of time but high enough to allow moves between different regions of the search space.

7 Conclusions

This article introduces the HSA to solve the WDND problem by combining Simulated Annealing with a local search [21]. HSA is evaluated using the three traditional cooling schemes (HSA_{Prop}, HSA_{Exp}, HSA_{Log}) and the proposed one in this work (HSA_{Rand}). Furthermore, other important control parameter, such as the initial temperature, is also considered. The performance of these HSA's variants is tested with 50 instances that come from 10 different HydroGen networks.

Regarding the comparison between the HSA's variants, the following conclusions are drawn. HSA_{Rand} that uses the random schedule process under any T_s, outperforms the remaining variants, because it achieves the best network designs in a reasonable execution times. In adittion, if the initial temperature is considered, the $T_s = 100$ allows to find the best TIC values by means of an appropriate balance between the exploration and exploitation of the search space.

When proposals are contrasted against ILS [21], the next remarks are obtained. The HSA's variants outperform ILS in the 80% of the WDND problem instances. Resulting HSA_{Rand} as the best option to solve this problem, in particular when $T_s = 100$.

A challenging extension of this work will be to propose and analyse different ways to set the length of the Markov Chain. We are also interested in testing larger dimension instances, as close as possible to real scenarios.

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