# MIXED-INTEGER LINEAR PROGRAMMING APPROACH FOR LIFE-CYCLE CARPET PROFIT

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**Abstract.** This paper proposes an mixed-integer linear programming (MILP) model to accurately represent a product life-cycle design considering profit maximization. The model that takes into account the effects on the demand level and a measure of the customer utility considering recycled raw materials and prices of the traditional and modular products. Demand functions for traditional and modular products are considered. Given the presence of bilinear terms in the formulation (for example due to the multiplication of product price for the demand), the multi-parametric disaggregation technique is used to obtain a linear model. The developed model is applied to a company that produces traditional carpets and it wants to manufacture carpets based on a new modular design where recycled materials must be incorporated. The objective of the company is to maximize the total profit taking into account the design specifications and the selling prices for traditional and modular carpets. In addition, the amount of square meters of traditional carpets must be determined and the take-back rate must be considered. The practical behavior of the formulation is analyzed through computational experiments exploring the analyzed case-study.

**Keywords:** Carpet recycling, product design, life-cycle, linear mathematical programming.

## 1 Introduction

The interest of manufacturers in recovering end-of-life products after customers use has markedly increased in the last years. The concern in natural resources, environmental government regulations, customers preferences, business environmental objectives and economic opportunities have been some of the most important reasons for the growing interest in considering end-of-life products. There are different options to deal with the recovered products, such as recycling, remanufacturing, reuse, repair, and refurbishment of components.

One relevant choice for companies is the recycling given that this activity preserves the raw materials incorporated to the initial production of goods allowing to manufacturers to increase the productivity as well as the profitability into the business. Nowadays, the potential of recycling is being exploded in different areas.

On sector where such problem is faced is the carpeting marked where a severe competition in the products industry is handled and where the recycling of end-of-life products is part important for manufacturers considering both economics and environmental conditions (Deutsche Umwelthilfe, 2017). The products recycling in charge of the carpet producers is a fundamental part to keep profitable the activity of the company and to fulfill certain environmental expectations. Thus, carpet manufacturers must design carpets taking into account reuse and recyclability, develop collection and recycling facilities and clearly label the materials used in the carpets to facilitate recycling by other actors.

Given the previously mentioned issues, the development of suitable computational tools for improving the life-cycle of products taking into account the designs and the profit is highly desirable. On the carpet sector some relevant papers addressing problems associated with the recycling network of carpets are Flapper et al. (1997) and Realff et al. (2004). Flapper et al. (1997) develop a deterministic location and capacity approach for the design of a recycling network for European carpet waste. Realff et al. (2004) develop a robust-MILP model to support decision-making for reverse production infrastructure design. Nevertheless, despite the importance of the carpet design for recycling activities, no work directly related to the mentioned topic has been found in the literature.

To the best of our knowledge, there is no previous research that proposes a formulation for the life-cycle design to maximize the total life-cycle profit of carpets considering different rates of recycling materials and demands for traditional and modular carpets.

## 2 Problem Description

The problem consists of optimizing the life-cycle profit of a company dedicated to carpet manufacturing. The company wants to incorporate a modern product designable to absorb partially or totally some materials that can be recovered from carpets built according to a traditional product design. The new product design is the company answer to new requirements in the care of the environment and the need to exploit economic opportunities.

The traditional product design (called Carpet Traditional Design: CTD) is based on a typical manufacturing process that maintains a certain demand level and the production is continued. Nevertheless, given that the concern in natural resources, environmental government regulations, customers' preferences, business environmental objectives and economic opportunities, a new product design is considered. The modern design is called: Carpet Eco-Modular Design (CEMD). This design allows incorporating as raw materials some resources that come from recovered carpets. In addition, the manufacture based in modules allows improving the separation and recovery of raw materials for future recycling life-cycles.

As part of the problem, the company must determine the prices of traditional and modular carpets, the production of both types of carpets, and the return rate of traditional products in order to allow the manufacture of modular products. In this case, it is considered that while the market demand of traditional carpets varies depending on the selling price, the market demand for modular carpets depends on the level of remanufactured products used and the price. The problem objective is to maximize the total life-cycle profit taking into account the sum of the profits from selling traditional and modular carpets.

After end-of-life traditional products are taken back, the carpets pass through reprocessing operations (Carpet Separation and Backing Shredding). The first step of reprocessing involves a hard process of separation of materials. Then, a part of the separated materials is shredded. The recycled process allows a) recovering materials that can be used as raw material for carpets (such as Nylon, PVC and mineral resources derived from Calcium) and b) generating products destined to shoddy for furniture stuffing and soil enhance. Figure 1 represents the main characteristics of the addressed problem. It is important to note that the Close-Loop Supply Chain (CLSC) design is out of the scope of this work and the network entities are illustrated in order to show the general context of the problem.

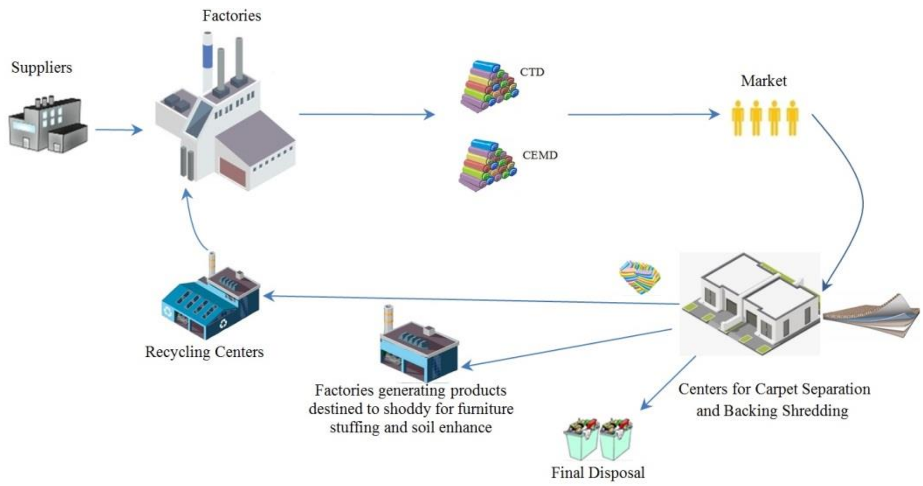


**Fig. 1.** Schematic representation of main characteristics of the problem

## 3 Formulation

The optimization of the life-cycle profit problem is formulated as an MILP model. The resulting approach arises of applying the multi-parametric disaggregation technique (Teles et al, 2011) for representing the bilinear terms that appear (for example due to the multiplication of price for the carpet demand) in order to maintain the model linearity. It is important to remark that bilinear terms are by definition non-convex and therefore these terms cannot be deal as any special case of quadratic programming. The problem was modeled as an MILP formulation in order to take advantage of the high efficiency, both in terms of solution accuracy and computing time, of the software tools (solvers) developed for MILPs. Therefore, this work introduces an MILP model that allows to accurately representing the life-cycle design while the profit is maximized and turn out to be computationally solvable.

In the formulation, the optimal selling prices, and the corresponding production strategies for both traditional and modular carpets are determined. The objective function of the approach is to maximize the sum of the profit from traditional carpets and the profit from selling products according to the CEMD using recycled materials. In particular, the paper presents a model that takes into account the effects on the demand level and a measure of the customer utility considering recycled raw materials and prices of traditional and modular products

The formulation is built according to the following assumptions:

a) The recovery level of PVC and mineral resources must be determined;

b) Recycled is instantaneous. Recycled operations have a negligible lead time;

c) Nylon can be destined to manufacture modular carpets or to third parties;

d) PVC and mineral resources can be destined to manufacture modular carpets or to third parties;

e) Shoddy for furniture stuffing and soil enhancer are destined to third parties;

f) The increase of recovery of PVC and mineral resources decreases the recovery of Shoddy for furniture stuffing and soil enhancer, respectively;

g) Parameters are assumed to be known and their estimation is outside the scope of this study;

h) Demand functions are considered as continuous linear functions of a single variable connected with the customer utility;

i) Customer utility function for traditional carpets is considered as a linear function of the selling price;

j) Customer utility function for modular carpets is considered as a linear function of the selling price and the level of use of recycled PVC and mineral resources.

To be more specific, the model takes into account the following decisions:

• the level of PVC and mineral resources recovered from the end-of-life traditional carpets.

• the design of modular carpets taking into account the level of recycled materials used and the selling price.

•the number square meters of traditional carpets that must be manufactured.

• the number of square meters of traditional carpets that must be taken back at the end-of-life stage.

• the number of square meters of modular carpets that must be manufactured.

Having in consideration the problem description, the mathematical formulation is presented next. The description starts with the objective function.

The objective function of the formulation (Equation 1) is to maximize the total life-cycle profit. The life-cycle profit is the sum of the profit from selling traditional carpets (PTC), and the profit from selling modular carpets (*PMC*) affected by a coefficient of discounted with an interest rate of θ.

$$Maximize \quad PTC + (1-\theta)^{-1}\, PMC \tag{1}$$

The computation of the profit for selling traditional carpets consists of three components: the revenue from selling $\beta T$ square meters of traditional carpets ($pT\beta T$), the cost of purchasing raw materials for making $\beta T$ square meters of traditional carpets (*CTP*), and the cost of assembling and distributing $\beta T$ square meters (*CTM*). Equation (2) is the linear representation of computation of the profit for selling traditional carpets. Since the profit for selling traditional carpets involves a non-convex bilinear term that arises from multiplying the selling price ($pT$) and the amount of products manufactured ($\beta T$), the multi-parametric disaggregation technique is used to achieve the model linearity. The linear version of the non-convex bilinear term according to the multi-parametric disaggregation technique is obtained including the Equations (3) – (8). The reformulation described by Teles et al. (2011) and Kolodziej et al. (2013) introduces auxiliary variables (binary and positive continuous variables: $zT_{klp}$ and $\widehat{pT}_{klp}$). Equations (3) and (4) are the discrete representation of variable $\beta T$ derived from generalized disjunctive programming (Grossmann and Ruiz, 2011). The representation of the value of $\beta T$ is discretized using the binary variable $zT_{klp}$ that selects one digit $k$ for each power $lp \in \mathbb{Z}$. Finally, Equations (5) - (8) allow the fully discretization of the bilinear product using the disaggregated continuous variable $\widehat{pT}_{klp}$. According to Equations (7) and (8), the variable $\widehat{pT}_{klp}$ must remain between the limits of the original variable $pT$ considering the values for the binary variable $zT_{klp}$, which are directly related with the $\beta T$ discretization. $pT_{Max}$ and $pT_{Min}$ are parameters considering the maximum and minimum values of the selling price of traditional carpets.

$$PTC = pT\beta T - CTP - CTM \tag{2}$$

$$\beta T = \sum_{lp\epsilon\mathbb{Z}} \sum_{k=0}^{9} 10^{lp} \cdot k \cdot zT_{klp} \tag{3}$$

$$\sum_{k=0}^{9} zT_{klp} = 1 \quad \forall\; lp \,\epsilon\, \mathbb{Z} \tag{4}$$

$$pT\beta T = \sum_{lp\epsilon\mathbb{Z}} \sum_{k=0}^{9} 10^{lp} \cdot k \cdot \widehat{pT}_{klp} \tag{5}$$

$$\sum_{k=0}^{9} \widehat{pT}_{klp} = pT \quad \forall\, lp \in \mathbb{Z} \tag{6}$$

$$\widehat{pT}_{klp} \leq pTMax \cdot zT_{klp} \quad \forall\, k \in K, \forall\, lp \in \mathbb{Z} \tag{7}$$

$$\widehat{pT}_{klp} \geq pTMin \cdot zT_{klp} \quad \forall\, k \in K, \forall\, lp \in \mathbb{Z} \tag{8}$$

Equations (9) and (10) determine the cost of purchasing raw material *i* for making *βT* square meters of carpets with design CTD (CTP). Equation (10) specifies the cost of assembling and distributing *βT* square meters of traditional carpets (CTM).$V_{ni}$ represents the market value of raw material *i*. $Req_{id}$ denotes the percentage of raw material i used for manufacturing carpets with design *d*. $cm_d$ indicates the cost of assembling and distributing each square meter of traditional carpets with design *d*. $Iin_d$ represents the set of raw materials used for manufacturing carpets with design *d*.

$$CTP = \sum_{i \in Iin_d} \beta T \cdot Vn_i \cdot Req_{id} \forall d \epsilon D \setminus \{CTD\} \tag{9}$$

$$CTM = cm_d \cdot \beta T \quad \forall d \epsilon D \setminus \{CTD\} \tag{10}$$

The computation of profit for selling modular products consists of six elements: the revenue from selling *βM* square meters of modular carpets (*pMβM*), the revenue from selling to third parties the recovered materials that are not used on modular carpets (*MR*), the cost of taking back *SR* square meters of end-of-life traditional carpets (*CRT*), the cost of acquiring raw materials for produces *βM* square meters of modular carpets (*CMP*), the cost of recovering materials (*CRR*) and the cost of assembling and distributing *βM* square meters of modular carpets (*CMM*). Equation (11) is the linear representation of computation of the profit for selling modular carpets. Newly, the computation of profit involves a non-convex bilinear term that arises from multiplying the selling price (*pM*) and the amount produced of modular carpets (*βM*). Therefore, in the same way it was done for traditional revenue, the multi-parametric disaggregation technique is also used in this case to achieve the linear version of the non-convex bilinear term. In this case, the linear form introduces the following auxiliary binary and positive continuous variables: $zM_{klp}$ and $\widehat{pM}_{klp}$.

$$PMC = pM\beta M + MR - CRT - CMP - CRR - CMM \tag{11}$$

Equation (12) represents the revenue from selling to third parties the materials recovered that are not used on modular carpets (*MR*). The revenue consists of two terms. The first term, implemented in Constraint (13), represents the revenue for selling recovered material *i* ($Rm_i$) to third parties given that these are not used for manufacturing modular carpets. The binary variable $y_i$ adopts the value 0 when the recovered material *i* is not used for manufacturing. The second term, implemented in Constraint (14), represents the revenue from selling recovered material *i* that exceed the amount required to produce the modular carpets. The condition of Constraint (14) is satisfied when $y_i$ =1 (the recovered material *i* is used for manufacturing) and the binary variable $l_i$ is equal to 0 given that recovered material *i* exceeds the amount of material required to produce the modular carpets. $Vm_i$ represents the market value of the recovered material *i*. *Iout* represents the set of materials that can be recovered from carpets.

$$MR = \sum_{i \in Iout} (MRug_i + MRe_i) \quad \textbf{(12)}$$

$$MRug_i \leq Vm_i \cdot Rm_i + BM \cdot (y_i) \quad \textbf{(13)}$$

$$MRe_i \leq Vm_i \cdot (Rm_i - (\beta M \cdot Req_{id})) + BM \cdot (1 - y_i) + BM \cdot (l_i) \qquad i \in Iout\ \forall\ d \in D\backslash\{CEMD\} \quad \textbf{(14)}$$

Equation (15) represents the cost of taking back *SR* square meters of end-of-life traditional carpets (*CRT*). *ct* indicates the cost of taking back each square meter of traditional carpets. Constraints (16) to (18) determine the cost of acquiring materials for producing *βM* square meters of modular carpets (*CMP*). Equation (17) represents the cost of purchasing material *i* when the material is upgraded. Equation (18) specifies the cost of purchasing material *i* when there is not enough recovered material *i* for manufacturing *βM* square meters of modular carpets.

$$CRT = ct \cdot SR \quad \textbf{(15)}$$

$$CMP = \sum_{i \in Iin_d,} (CMPu_i + CMPr_i) \qquad \forall\ d \in D\backslash\{CEMD\} \quad \textbf{(16)}$$

$$CMPu_i \geq Vn_i \cdot \beta M \cdot Req_{id} - BM \cdot (y_i) \qquad \forall\ i \in Iin_d, \quad \forall\ d \in D\backslash\{CEMD\} \quad \textbf{(17)}$$

$$CMPr_i \geq Vn_i \cdot (\beta M \cdot Req_{id} - Rm_i) - BM \cdot (1 - y_i) - BM \cdot (1 - l_i) \qquad \forall\ i \in Iin_d, \quad \forall\ d \in D\backslash\{CEMD\} \quad \textbf{(18)}$$

Equations (19)-(21) represent the cost of recovering material (*CRR*). Equation (20) represents the cost of recovering material *i* that is used on the modular carpets. $cr_i$ indicates the cost of recovering material *i* from each square meter of traditional carpets. Equation (21) specifies the cost of recovering material *i* required to meet the production of modular carpets. Finally, equation (22) represents the cost of assembling and distributing $\beta M$ square meters of modular carpets ($CMM$).

$$CRR = \sum_{\forall\, i\, \epsilon\, Iout} (CRRo_i + CRRa_i) \quad \textbf{(19)}$$

$$CRRo_i \geq\ cr_i \cdot Rm_i -\ BM \cdot (1 - y_i) -\ BM \cdot (1 - l_i)\ \ \forall\, i\, \epsilon\, Iout \quad \textbf{(20)}$$

$$CRRa_i \geq\ cr_i \cdot \beta M \cdot Req_{id} -\ BM \cdot (1 - y_i) -\ BM \cdot (l_i) \quad \forall\, i\, \epsilon\, Iout, \forall\, d\, \epsilon D \backslash \{CEMD\} \quad \textbf{(21)}$$

$$CMM = cm_d \cdot \beta M \forall d \epsilon D \backslash \{CEMD\} \quad \textbf{(22)}$$

Equation (23), which describes the supply of end-of-life products ($SR$),is the linear representation of the non-convex bilinear term that arises from multiplying the take-back rate ($\alpha$) and the amount of square meters of traditional carpets manufactured ($\beta T$). Newly, the multi-parametric disaggregation technique is used to achieve the model linearity. The linear version of the non-convex bilinear term according to the multi-parametric disaggregation technique is obtained considering Equations (3), (4) and (24)–(27). The reformulation introduces the positive continuous variable $\hat{\alpha}_{klp}$ and used the binary variable $zT_{klp}$. Finally, equations (24)-(27) permit the discretization of the bilinear product using the disaggregated continuous variable $\hat{\alpha}_{klp}$. According to equations (26) and (27), variable $\hat{\alpha}_{klp}$ must remain between the limits of the original variable ($\alpha$) taking into account the values for the binary variable $zT_{klp}$. $\alpha Max$ and $\alpha Min$ are parameters considering the maximum and minimum take-back rate of end-of-life traditional carpets.

$$SR = \alpha\beta T \quad \textbf{(23)}$$

$$\alpha\beta T = \sum_{lp \epsilon \mathbb{Z}} \sum_{k=0}^{9} 10^l \cdot k\ \cdot \hat{\alpha}_{klp} \quad \textbf{(24)}$$

$$\sum_{k=0}^{9} \hat{\alpha}_{klp} =\ \alpha \quad \forall\, lp\, \epsilon\, \mathbb{Z} \quad \textbf{(25)}$$

$$\hat{\alpha}_{klp} \leq\ \alpha Max\ \cdot zT_{klp}\ \ \forall\, k\, \epsilon\, K, \forall\, lp\, \epsilon\, \mathbb{Z} \quad \textbf{(26)}$$

$$\hat{\alpha}_{klp} \geq \alpha Min\ \cdot zT_{klp} \quad \forall\, k\, \epsilon\, K, \forall\, lp\, \epsilon\, \mathbb{Z} \quad \textbf{(27)}$$

Equation (28) imposes a given value to the rate of certain recovery materials ($mr_i$) belonging to the set *Ioutfmr*. *Ioutfmr* represents the set of materials that are recovered from carpets at a given fixed rate. Constraints (29) and (30) bound the maximum and minimum values that can be adopted by recovery rate of materials belonging to the set *Ioutvmr*. *Ioutvmr* represents the set of materials that are recovered from carpets at a given variable rate. Thus, since the recovery levels of mineral resources and PVC affect the levels of soil enhancer and shoddy sold to third parties, respectively, Constraint (31) imposes the condition of equality to the maximum value allowed for the recovery rates ($rMaxi_{ii'}$). *IIout* set of pairs of materials *i* and *i`* where their recovery levels depend on each other, ($i,i` \in Iout,\ i \neq i`$).

Equation (32) determines the amount of material *i* available for being used for modular carpets or for being sold to third parties ($Rm_i$) in function of the recovery rate $mr_i$ and the supply of the end-of-life traditional carpets (*SR*). Equation (33) takes into account whether the amount of reusable material *i* ($Rm_i$) is enough to produce the required amount of square meters of modular carpets (*βM*). Thus, if the amount of recovered material *i* is insufficient in quantity to the required amount to manufacture *βM* square meters of modular carpets, the binary variable $l_i$ is equal to 1.

$$mr_i = fmr_i \forall\ i \in Ioutfmr \tag{28}$$

$$mr_i \leq fmrMax_i \forall\ i \in Ioutvmr \tag{29}$$

$$mr_i \geq fmrMin_i \forall\ i \in Ioutvmr \tag{30}$$

$$mr_i + mr_{i`} = rMaxi_{ii'} \forall\ i, i` \in IIout \tag{31}$$

$$Rm_i = mr_i \cdot SR \forall\ i \in Iout \tag{32}$$

$$(\beta M \cdot Req_{id} - Rm_i) \leq \ BM \cdot l_{it} \quad \forall\ i \in Iout, \quad \forall\ d \in D \backslash \{CEMD\} \tag{33}$$

Equations (34) and (35) are expressions of the customer utility functions for traditional and modular carpets, respectively. In this work, it is considered that the utility functions for traditional and modular carpets are defined as linear weighted sums of the selling prices(*pT* and *pM*) and the recovery rate $mr_i$ of the set of materials that can be recovered (*Ioutm*: set of materials that can be recovered for manufacturing modular carpets). Parameters *wpt*, *wpM* and $wM_i$, are weight factors to adjust the importance of the prices and the manufacturing characteristics of modular carpets.

$$UN = wpT \cdot \left(1 - \frac{pT}{pTMax}\right) \tag{34}$$

$$UM = \sum_{\forall\ i \in Ioutm} wM_i \cdot \left(1 - \frac{mr_i}{fmrMax_i}\right) + wpM \cdot \left(1 - \frac{pM}{pMMax}\right) \tag{35}$$

Equation (36) constrains the production of square meters of traditional carpets (*βT*) to be less than or equal to the demand size (*DT*). In addition, Equations (37) and (38) limit the production of modular carpets (*βM*) to be less than or equal to the demand size (*DM*) and the supply of the end-of-life product (*SR*), respectively.

$$\beta T \leq DT \qquad \textbf{(36)}$$

$$\beta M \leq DM \qquad \textbf{(37)}$$

$$\beta M \leq SR \qquad \textbf{(38)}$$

Equations (39) and (40) are generic expressions of the demands for traditional and modular carpets, respectively. In this work, it is assumed that the values of the utility functions *UT* and *UM* determines the demands (*DT* and *DM*) in function of markets size for traditional and modular carpets (*QT* and *QM*). Constraint (41) limits the price of modular carpets (*pM*) to be less than or equal to the price of traditional carpets (*pT*).

$$DT = fT(UT) \qquad \textbf{(39)}$$

$$DM = fM(UM) \qquad \textbf{(40)}$$

$$pM \leq pT \qquad \textbf{(41)}$$

## 4 Case study

Figure 2 shows the raw materials and the resources that can be recovered after the end-of-life of each carpet design. For example, given that the traditional carpet recycling process is inefficient, the PVC and mineral resources are recovered in low proportions (at most 12% and 6% of the 29.4% and 39.4% of PVC and mineral resources used per square meter of carpet, respectively). However, Nylon is almost completely recovered (15% of the 17.2% used per square meter of traditional carpet).

In this work, the demands for traditional and modular carpets are considered as linear functions of the sizes of the markets (*QT*=1200000$m^2$ and *QM*=900000$m^2$) and measures of customer utilities for both carpets types (*UT* and *UM*). The measures for customer utilities are defined as functions of the level of use of recycled materials and the selling prices.

It is expected that the sales of traditional carpets will be gradually available for buy-back during a period of 10 years of use, and, in the same period of 10 years, the company is gradually manufacturing modular carpets using the recovered carpets. The interest rate for a period of time of 10 years is assumed to be 3% ($\theta = 0.03$). Maximum selling prices are *pTMax*=\$90 and *pMMax*=\$65.

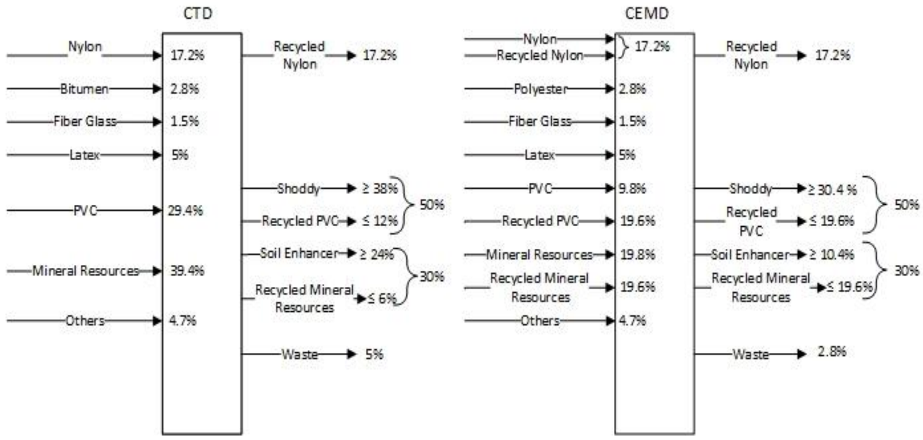


**Fig. 2.** Raw materials requirements and possible recovery of resources according to both, traditional and modular designs of carpets

## 5 Computational Results

The proposed linear model was coded in GAMS (release 25.1.1) optimizer software, and all computations were run with CPLEX 12.8, on a HP Z800 workstation with Intel Xeon x5650 2.66 GHz and 16 GB RAM memory for a 0.1% gap tolerance. The total solution CPU time was 20seconds.The multi-parametric disaggregation representation uses as lowest and largest power of the decimal base, the numbers 0 and 7 ($lp$={0,..,7}) given the minimum and maximum values that can achieve the production of carpets ($0 \leq \beta T \leq 1200000$ and $0 \leq \beta M \leq 900000$).The linear programming formulation applied to the problem contains 188 binary variables, 1843 continuous variables and 3032 constraints.

The total profit taking into account the design specifications and the selling prices for traditional and modular carpets is $49.46 million. The optimal selling price for a square meter for both traditional and modular carpets is $45.67. The total profit expected from selling 590990 $m^2$ of modular carpets is approximately $ 26.99 million. Thus, the manufacturing of modular carpets is profitable.

To take advantage of the economic opportunity of recycling materials, the company should take back all the traditional carpets manufactured ($\alpha$=1then $SR$= 590990 $m^2$).The optimal recycling rate ($mr_i$) for Nylon, PVC and mineral resources are 15%, 2.7% and 1.3%, respectively. The modest rate of using PVC and mineral resources in modular carpets is given to the trade-off between the use of recovered materials and the price of modular carpets.

It is important to note that while the total cost of purchasing raw materials for making $\beta T$ $m^2$ of carpets with design *CTM* is $478613, the cost for producing $\beta M$ $m^2$ of carpets with design *CEMD* is $264969. Thus, it can be seen that the recycling of Nylon, PVC and mineral resources (mainly Nylon recycling), substantially reduces the costs of purchasing raw materials.

## 6 Conclusion

The proposed optimization approach can serve as a suitable tool for improving the life-cycle profit taking into account the carpet designs, especially the modular design where the levels of recycled materials and the recovery rates are determined. The formulation provides information about the selling prices and the demands to be satisfied with both, traditional and modular carpets. In addition, the model determines the materials that must be used for manufacturing modular carpets or be sold to third parties. Finally, the information obtained allows understanding the path of recovered materials.

The resulting MILP model arises of applying multi-parametric disaggregation technique for representing the bilinear terms of the problem. It is important to note that the objective of the resultant linear formulation is to obtain a problem representation able to be solved to global optimality. Thus, the problem modeled as a mixed-integer linear formulation can take advantage of the high efficiency, both in terms of solution accuracy and computing time of the software tools (solvers) developed for MILP.

In the future, the model should be improved to address a planning horizon with at least two time periods in order to evaluate the effects in manufacturing activities of the recovery of end-of-life traditional and modular carpets.

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